Glenbard East High School

Summer Review Packet

For Students entering PRE-CALCULUS

Name:______________________________________

1. This packet is to be handed in to your Pre-Calculus teacher on the first day of the school year.
2. All work must be shown in the packet OR on a separate sheet of paper attached to the packet.
3. Completion of this packet will be worth a grade and will be recorded first semester.
4. If you successfully complete Summer Bridges for Pre-Calculus, you may be exempt from completion of this packet. *Have your Summer Bridges teacher notify your Pre-Calculus teacher upon successful completion of Bridges.*
5. Answers to odd-numbered problems have been provided.

My signature below indicates that I have received the Pre-Calculus Summer Review Packet.

______________________________________ ________________________

(Student Signature) **(Date)**

Radicals:

To simplify means that 1) no radicand has a perfect square factor and

2) there is no radical in the denominator (rationalize).

Recall the **Product Property** $\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \bullet \sqrt{6}$ find the perfect square factor

$= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}}$ multiply numerator & denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms, multiply the numerator and denominator by conjugate of the denominator (the conjugate of $3+\sqrt{2}$ is $3-\sqrt{2}$)

Simplify each of the following.

1. $\sqrt{32}$
2. $\sqrt{(2x)^8}$
3. $\sqrt[3]{-64}$
4. $\sqrt{49m^2n^8}$
5. $\sqrt{\frac{11}{9}}$
6. $\sqrt{60} \bullet \sqrt{105}$
7. $\left(\sqrt{5}-\sqrt{6}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Rationalize.

8. $\frac{1}{\sqrt{2}}$

9a. $\frac{2}{\sqrt{3}}$

10a. $\frac{3}{2-\sqrt{5}}$

Complex Numbers:

Form of complex number: $a+bi$

Where *a* is the real part and the *bi* is the imaginary part

Always make these substitutions $\sqrt{-1}=i$ and $i^2=-1$

To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5}=\sqrt{-1}\bullet\sqrt{5}$ Pull out $\sqrt{-1}$

$= i\sqrt{5}$ Make substitution

Example: $\left(i\sqrt{5}\right)^2=i\sqrt{5}\bullet i\sqrt{5}$

$=i^2\sqrt{25}$ $=(-1)(5)=-5$

Treat *i* like any other variable when $+,-,\times, or \div$ (but always simplify $i^2=-1$)

Example: $2i(3+i)=2(3i)+2i(i)$ Distribute

$=6i+2i^2$ Simplify

$=6i+2(-1)$ Substitute

$=-2+6i$ Simplify and rewrite in complex form

Since $i=\sqrt{-1}$, no answer can have an 'i' in the denominator. RATIONALIZE!

Simplify.

9b. $\sqrt{-49}$

10b. $6\sqrt{-12}$

11. $-6(2-8i)+3(5+7i)$

12. $(3-4i)^2$

13. $(6-4i)(6+4i)$

Rationalize.

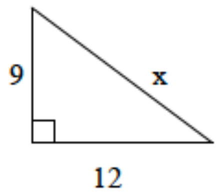
14. $\dfrac{1+6i}{5i}$

Geometry:

Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Find the value of x.

15.



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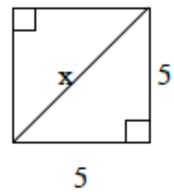

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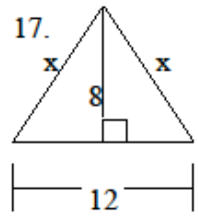
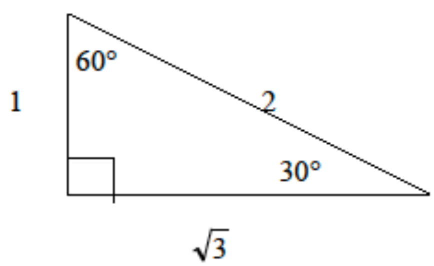
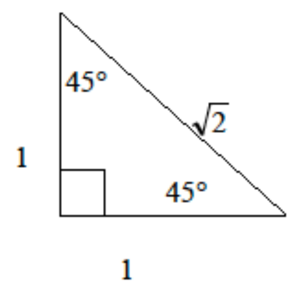


18. A square has perimeter 12 cm. Find the length of the diagonal.

* In $30° - 60° - 90°$ triangles,
sides are in proportion $1, \sqrt{3}, 2$.

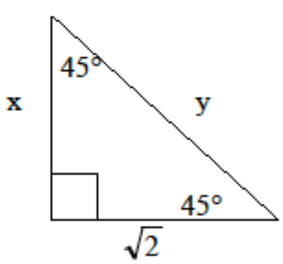


*In $45° - 45° - 90°$ triangles,
sides are in proportion $1, 1, \sqrt{2}$.



Solve for x and y.

19.



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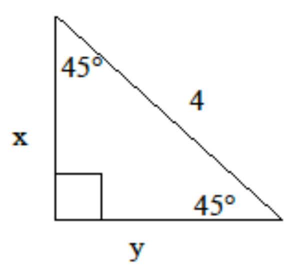

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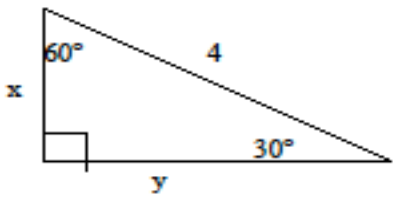

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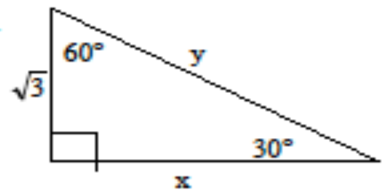


Equations of Lines:	
Slope-intercept form: $y = mx + b$	Vertical line: $x = c$ *(slope is undefined)*
Point-slope form: $y - y_1 = m(x - x_1)$	Horizontal line: $y = c$ *(slope is zero)*
Standard Form: $Ax + By = C$	Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equation: $5x - 4y = 8$

24. Find the x-intercept and y-intercept of the equation: $2x - y = 5$

25. Write the equation in standard form: $y = 7x - 5$

Write the equation of the line in slope-intercept form with the following conditions:

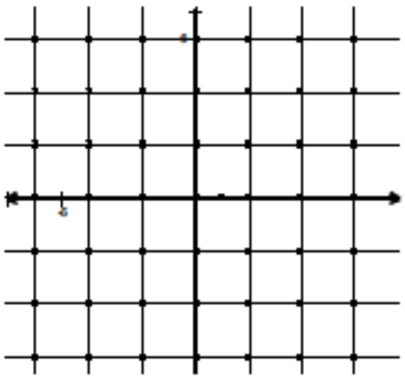
26. slope = -5 and passes through the point (-3, -8)

27. passes through the points (4, 3) and (7, -2)

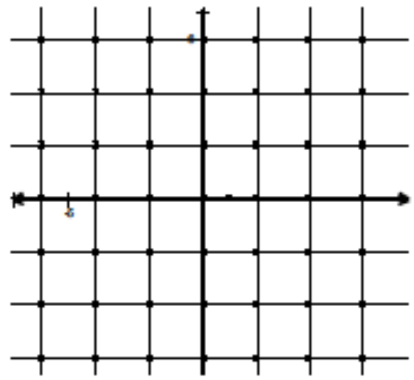
28. x-intercept = 3 and y-intercept =2

Graphing: **Graph each function, inequality, and/or system.**

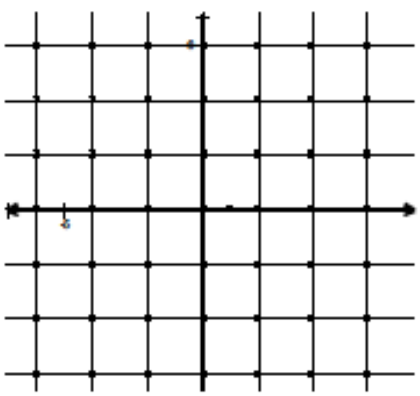
29. $3x - 4y = 12$



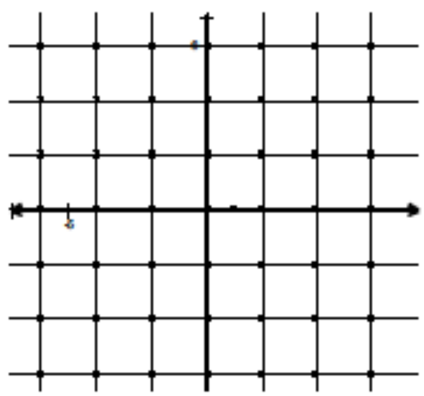
30. $\begin{cases} 2x + y = 4 \\ x - y = 2 \end{cases}$



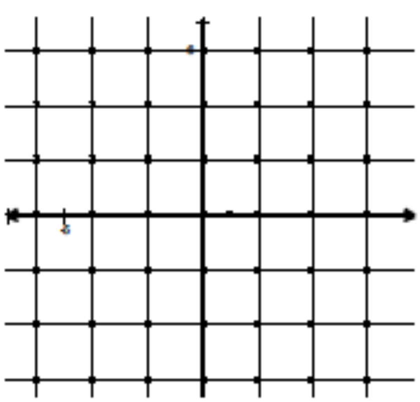
31. $y < -4x - 2$



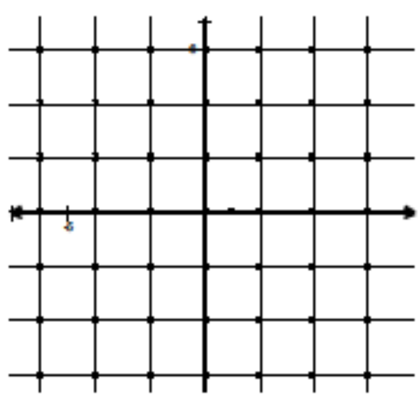
32. $y + 2 = |x + 1|$



33. $y > |x| - 1$



34. $y + 4 = (x - 1)^2$



Systems of Equations:

$$\begin{cases} 3x + y = 6 \\ 2x - 2y = 4 \end{cases}$$

Substitution:	**Elimination:**
Solve 1 equation for 1 variable	Find opposite coefficients for 1 variable
Rearrange.	Multiply equation(s) by constant(s).
Plug into 2nd equation.	Add equations together (lose 1 variable)
Solve for the other variable.	Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$y = 6 - 3x$	Solve 1st equation for y	$6x + 2y = 12$	Multiply 1st equation by 2
$2x - 2(6 - 3x) = 4$	Plug into 2nd equation	$\underline{2x - 2y = 4}$	coefficients of y are opposite
$2x - 12 + 6x = 4$	Distribute	$8x = 16$	Add
$8x = 16$ and $x = 2$	Simplify	$x = 2$	Simplify.

Plug x=2 back into the original equation $\begin{array}{l} 6 + y = 6 \\ y = 0 \end{array}$

Solve each system of equations, using any method.

35. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

36. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Exponents:

Recall the following rules of exponents:

1.	$a^1 = a$	Any number raised to the power of one equals itself.
2.	$1^a = 1$	One raised to any power is one.
3.	$a^0 = 1$	Any nonzero number raised to the power of zero is one.
4.	$a^m \cdot a^n = a^{m+n}$	When multiplying two powers that have the same base, add the exponents.
5.	$\dfrac{a^m}{a^n} = a^{m-n}$	When dividing two powers with the same base, subtract the exponents.
6.	$(a^m)^n = a^{mn}$	When a power is raised to another power, multiply the exponents.
7.	$a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$	Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\dfrac{3c}{c^{-1}}$

40. $\dfrac{2ef^{-1}}{e^{-1}}$

41. $\dfrac{\left(n^3p^{-1}\right)^2}{\left(np\right)^{-2}}$

Simplify.

42. $3m^2 \bullet 2m$

43. $\left(a^3\right)^2$

44. $(-b^3c^4)^5$

45. $4m(3a^2m)$

Polynomials:

To add/subtract polynomials, combine like terms.

EX: $8x-3y+6-(6y+4x-9)$ *Distribute the negative through the parantheses.*

$=8x-3y+6-6y-4x+9$ *Combine like terms with similar variables.*

$=8x-4x-3y-6y+6+9$

$=4x-9y+15$

Simplify.

46. $3x^3+9+7x^2-x^3$

47. $7m-6-(2m+5)$

To multiply two binomials, use FOIL.

EX: $(3x-2)(x+4)$ *Multiply the first, outer, inner, and last terms.*

$=3x^2+12x-2x-8$ *Combine like terms together.*

$=3x^2+10x-8$

Multiply.

48. $(3a+1)(a-2)$

49. $(s+3)(s-3)$

50. $(c-5)^2$

51. $(5x+7y)(5x-7y)$

Factoring:

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

a) If you have one (other than 1) factor it out.

b) If you don't have one move on to STEP 2

STEP 2: How many terms does the polynomial have?

2 Terms a) is it the difference of two squares? $a^2 - b^2 = (a+b)(a-b)$

EX: $x^2 - 25 = (x+5)(x-5)$

b) Is it the sum or difference of two cubes? $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

EX: $m^3 + 64 = (m+4)(m^2-4m+16)$
$p^3 - 125 = (p-5)(p^2+5p+25)$

3 Terms	**EX:**
$x^2+bx+c=(x+\_)(x+\_)$	$x^2+7x+12=(x+3)(x+4)$
$x^2-bx-c=(x-\_)(x-\_)$	$x^2-5x+4=(x-1)(x-4)$
$x^2+bx-c=(x-\_)(x+\_)$	$x^2+6x-16=(x-2)(x+8)$
$x^2-bx-c=(x-\_)(x+\_)$	$x^2-2x-24=(x-6)(x+4)$

4 Terms---Factor by Grouping

a) Pair up first two terms and last two terms.

b) Factor out GCF of each pair of numbers.

c) Factor out front parentheses that the terms have in common.

d) Put leftover terms in parentheses.

$$Ex: x^3+3x^2+9x+27 = \left(x^3+3x^2\right)+\left(9x+27\right)$$
$$= x^2(x+3)+9(x+3)$$
$$= (x+3)\left(x^2+9\right)$$

Factor completely.

52. $z^2+4z-12$

53. $6-5x-x^2$

54. $2k^2+2k-60$

55. $-10b^4-15b^2$

56. $9c^2+30c+25$

57. $9n^2-4$

58. $27z^3-8$

59. $2mn-2mt+2sn-2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use the quadratic formula.

EX: $x^2-4x=21$ *Set equal to zero FIRST.*

$x^2-4x-21=0$ *Now factor.*

$(x+3)(x-7)=0$ *Set each factor equal to zero.*

$x+3=0 \quad x-7=0$ *Solve for each x.*

$x=-3 \quad x=7$

Solve each equation.

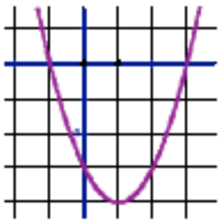
60. $x^2-4x-12=0$

61. $x^2+25=10x$

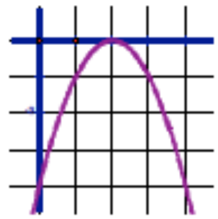
62. $x^2-14x+40=0$

Discriminant: The number under the radical in the quadratic formula $(b^2 - 4ac)$ can tell you what kind of roots you will have.

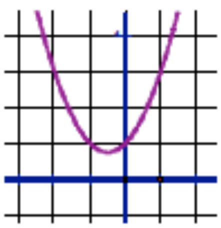
If $b^2 - 4ac > 0$ you will have TWO real roots (touches the x-axis twice)



If $b^2 - 4ac = 0$ you will have ONE real root (touches axis once)



If $b^2 - 4ac < 0$ you will have TWO imaginary roots. (Function does not cross the x-axis)



QUADRATIC FORMULA—allows you to solve any quadratic for all its real and imaginary roots.

$5x^2 - 2x + 4 = 0$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EX: In the equation $x^2 + 2x + 3 = 0$, find the value of the discriminant, describe the nature of the roots, then solve.

$x^2 + 2x + 3 = 0$ *Determine the values of a, b, and c.*

$a = 1$ $b = 2$ $c = 3$ *Find the discriminant.*

$D = 2^2 - 4 \cdot 1 \cdot 3$

$D = 4 - 12$

$D = -8$ *There are two imaginary roots.*

$Solve:\ x = \dfrac{-2 \pm \sqrt{-8}}{2}$

$x = \dfrac{-2 \pm 2i\sqrt{2}}{2}$

$x = -1 \pm i\sqrt{2}$

Find the value of the discriminant, describe the nature of the roots, then solve each quadratic. Use EXACT values.

63. $x^2 - 9x + 14 = 0$

Discriminant = ____________________________

Type of Roots: ____________________________

Exact Value of Roots: ____________________

64. $5x^2 - 2x + 4 = 0$

Discriminant = __________________________________

Type of Roots: __________________________________

Exact Value of Roots: __________________________

<u>Long Division</u>—can be used when dividing any polynomials.

Synthetic Division—can ONLY be used when dividing a polynomial by a linear polynomial.

EX: $\dfrac{2x^3+3x^2-6x+10}{x+3}$

Long Division

$$\frac{2x^3+3x^2-6x+10}{x+3}$$

$$\begin{array}{r} 2x^2-3x+3+\dfrac{1}{x+3} \\ x+3\overline{\big)\;2x^3+3x^2-6x+10} \\ (-)\left(2x^3+6x^2\right) \\ -3x^2-6x \\ (-)\left(-3x^2-9x\right) \\ 3x+10 \\ (-)\;(3x+9) \\ 1 \end{array}$$

Synthetic Division

$$\frac{2x^3+3x^2-6x+10}{x+3}$$

$$\frac{2x^3+3x^2-6x+10}{x+3}$$

$$\begin{array}{rrrrr} -3 & 2 & 3 & -6 & 10 \\ & & -6 & 9 & -9 \\ & 2 & -3 & 3 & 1 \end{array}$$

$$=2x-3x+3+\frac{1}{x+3}$$

Divide each polynomial using long division OR synthetic division.

65. $\dfrac{c^3-3c^2+18c-16}{c^2+3c-2}$

66. $\dfrac{x^4-2x^2-x+2}{x+2}$

To evaluate a function for the given value, simply plug the value into the function for x.

Evaluate each function for the given value.

67. $f(x) = x^2 - 6x + 2$

$f(3) =$ __________

68. $g(x) = 6x - 7$

$g(x+h) =$ __________

69. $f(x) = 3x^2 - 4$

$5\left[f\left(x+2\right)\right] =$ __________

Composition and Inverses of Functions:

Recall: $(f \circ g)(x) = f(g(x))$ OR $f\left[g(x)\right]$ read "**f of g of x**" means to plug the inside function in for x in the outside function.

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find $f(g(x))$.

$$
\begin{aligned}
f(g(x)) &= f(x-4) \\
&= 2(x-4)^2 + 1 \\
&= 2(x^2 - 8x + 16) + 1 \\
&= 2x^2 - 16x + 32 + 1 \\
f(g(x)) &= 2x^2 - 16x + 33
\end{aligned}
$$

Suppose $f(x) = 2x,\ g(x) = 3x - 2,\ and\ h(x) = x^2 - 4$. **Find the following:**

70. $f\left[g(2)\right] =$ ________

71. $f\left[g(x)\right] =$ __________

72. $f\left[h(3)\right] =$ ________

73. $g\left[f(x)\right] =$ __________

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Example:	$f(x) = \sqrt[3]{x+1}$	Rewrite *f(x)* as *y*
	$y = \sqrt[3]{x+1}$	Switch *x* and *y*
	$x = \sqrt[3]{y+1}$	Solve for your new *y*
	$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
	$x^3 = y+1$	Simplify
	$y = x^3 - 1$	Solve for *y*
	$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse, $f^{-1}(x)$, if possible.

74. $f(x) = 5x + 2$

75. $f(x) = \frac{1}{2}x - \frac{1}{3}$

Rational Algebraic Expressions:

Multiplying and Dividing: **Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.**

EX: $\dfrac{x^2+10x+21}{5-4x-x^2}\bullet\dfrac{x^2+2x-15}{x^3+4x^2-21x}$ *Factor everything completely.*

$=\dfrac{(x+7)(x+3)}{(5+x)(1-x)}\bullet\dfrac{(x+5)(x-3)}{x(x-3)(x+7)}$ *Cancel out common factors in the top and bottom.*

$=\dfrac{(x+3)}{x(1-x)}$ *Simplify.*

76. $\dfrac{5z^3+z^2-z}{3z}$

77. $\dfrac{m^2-25}{m^2+5m}$

78. $\dfrac{10r^5}{21s^2}\bullet\dfrac{3s}{5r^3}$

79. $\dfrac{a^2-5a+6}{a+4}\bullet\dfrac{3a+12}{a-2}$

80. $\dfrac{6d-9}{5d+1}\div\dfrac{6-13d+6d^2}{15d^2-7d-2}$

<u>Addition and Subtraction</u>

First find the least common denominator. Write each fraction with that LCD. Add/subtract numerators as indicated and leave the denominators as they are.

EX:	$\frac{3x+1}{x^2+2x}+\frac{5x-4}{2x+4}$	*Factor denominator completely.*
	$\frac{3x+1}{x(x+2)}+\frac{5x-4}{2(x+2)}$	*Find LCD, which is* $(2x)(x+2)$
	$\frac{2(3x+1)}{2x(x+2)}+\frac{x(5x-4)}{2x(x+2)}$	*Rewrite each fraction with the LCD in the denominator.*
	$\frac{6x+2+5x^2-4x}{2x(x+2)}$	*Write as one fraction.*
	$\frac{5x^2+2x+2}{2x(x+2)}$	*Combine like terms.*

81. $\frac{2x}{5}-\frac{x}{3}$

82. $\frac{b-a}{a^2b}+\frac{a+b}{ab^2}$

83. $\frac{2-a^2}{a^2+a}+\frac{3a+4}{3a+3}$

<u>Complex Fractions:</u> Eliminate complex fractions by multiplying the numerator and denominator by the LCD of each of the small fractions. Then simplify the result.

EX: $\dfrac{1+\dfrac{1}{a}}{\dfrac{2}{a^2}-1}$ *Find LCD:* a^2

$= \dfrac{\left(1+\dfrac{1}{a}\right)\bullet a^2}{\left(\dfrac{2}{a^2}-1\right)\bullet a^2}$ *Multiply top and bottom by LCD.*

$= \dfrac{a^2+a}{2-a^2}$ *Factor and simplify if possible.*

$= \dfrac{a(a+1)}{2-a^2}$

84. $\dfrac{1-\dfrac{1}{2}}{2+\dfrac{1}{4}}$

85. $\dfrac{1+\dfrac{1}{z}}{z+1}$

86. $\dfrac{5+\dfrac{1}{m}-\dfrac{6}{m^2}}{\dfrac{2}{m}-\dfrac{2}{m^2}}$

87. $\dfrac{2+\dfrac{1}{x}-\dfrac{1}{x^2}}{1+\dfrac{4}{x}+\dfrac{3}{x^2}}$

Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of our fractions. Then solve the equation as usual.

$$\frac{5}{x+2}+\frac{1}{x}=\frac{5}{x}$$ *Find LCD first* $x(x+2)$

$$x(x+2)\frac{5}{x+2}+x(x+2)\frac{1}{x}=\frac{5}{x}x(x+2)$$ *Multiply each term by the LCD.*

$$5x+1(x+2)=5(x+2)$$ *Simplify and solve.*

$$5x+x+2=5x+10$$

$$6x+2=5x+10$$

$x=8$ ← *Check your answer! Sometimes they do not check!*

Check:

$$\frac{5}{8+2}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{5}{10}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{5}{8}=\frac{5}{8}$$

Solve each equation. Check your solutions.

88. $\frac{12}{x}+\frac{3}{4}=\frac{3}{2}$

89. $\frac{x+10}{x^2-2}=\frac{4}{x}$

90. $\frac{5}{x-5}=\frac{x}{x-5}-1$

Summer Review Packet for Pre-Calculus

Solutions to Odd Exercises

1. $4\sqrt{2}$ **3.** -4 **5.** $\frac{\sqrt{11}}{3}$ **7.** $5+\sqrt{10}-\sqrt{30}-2\sqrt{3}$ **9a.** $\frac{2\sqrt{3}}{3}$

9b. $7i$ **11.** $3+69i$ **13.** 52 **15.** 15 **17.** 10 **19.** $\sqrt{2}$

21. *x=2* $y=2\sqrt{3}$ **23.** $m=\frac{5}{4}\quad b=-2$ **25.** $7x-y=5$ **27.** $y=-\frac{5}{3}x+\frac{29}{3}$

29, 31, 33 graphs **35.** (7, -10) **37.** $\left(\frac{89}{36},-\frac{29}{18}\right)$ **39.** $3c^2$ **41.** n^8

43. a^6 **45.** $12a^2m^2$ **47.** $5m-11$ **49.** s^2-9

51. $25x^2-49y^2$ **53.** $-(x+6)(x-1)$ **55.** $-5b^2(2b^2+3)$ **57.** $(3n-2)(3n+2)$

59. $2(m+s)(n-t)$ **61.** $x=5$ **63.** $25;\ 2\ \ real;\ 7\ \ and\ \ 2$ **65.** $(c-6)+\frac{38c-28}{c^2+3c-2}$

67. -7 **69.** $15x^2+60x+40$ **71.** $6x-4$ **73.** $6x-2$

75. $f^{-1}(x)=2x+\frac{2}{3}$ **77.** $\frac{m-5}{m}$ **79.** $3(a-3)$ **81.** $\frac{x}{15}$ **83.** $\frac{4a+6}{3a(a+1)}$

85. $\frac{1}{z}$ **87.** $\frac{2x-1}{x+3}$ **89.** $x=4,-\frac{2}{3}$