

Glenbard East High School

Summer Review Packet

For Students entering PRE-CALCULUS

Name: _____

1. This packet is to be handed in to your Pre-Calculus teacher on the first day of the school year.
2. All work must be shown in the packet OR on a separate sheet of paper attached to the packet.
3. Completion of this packet will be worth a quiz grade and will be recorded for the first quarter.
4. If you successfully complete Summer Bridges for Pre-Calculus, you may be exempt from completion of this packet. *Have your Summer Bridges teacher notify your Pre-Calculus teacher upon successful completion of Bridges.*
5. Answers to odd-numbered problems have been provided.

My signature below indicates that I have received the Pre-Calculus Summer Review Packet.

(Student Signature)

(Date)

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Radicals:

To simplify means that 1) no radicand has a perfect square factor and

2) there is no radical in the denominator (rationalize).

Recall the **Product Property** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$ find the perfect square factor

$$= 2\sqrt{6} \quad \text{simplify}$$

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ multiply numerator & denominator by $\sqrt{2}$

$$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2} \quad \text{multiply straight across and simplify}$$

If the denominator contains 2 terms, multiply the numerator and denominator by conjugate of the denominator (the conjugate of $3 + \sqrt{2}$ is $3 - \sqrt{2}$)

Simplify each of the following.

1. $\sqrt{32}$

2. $\sqrt{(2x)^8}$

3. $\sqrt[3]{-64}$

4. $\sqrt{49m^2n^8}$

5. $\sqrt{\frac{11}{9}}$

6. $\sqrt{60} \cdot \sqrt{105}$

7. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

Rationalize.

8. $\frac{1}{\sqrt{2}}$

9a. $\frac{2}{\sqrt{3}}$

10a. $\frac{3}{2 - \sqrt{5}}$

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Complex Numbers:

Form of complex number: $a + bi$

Where a is the real part and the bi is the imaginary part

Always make these substitutions $\sqrt{-1} = i$ and $i^2 = -1$

To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5} = \sqrt{-1} \cdot \sqrt{5}$ Pull out $\sqrt{-1}$ Example: $(i\sqrt{5})^2 = i\sqrt{5} \cdot i\sqrt{5}$
 $= i\sqrt{5}$ Make substitution $= i^2 \sqrt{25} = (-1)(5) = -5$

Treat i like any other variable when $+$, $-$, \times , or \div (but always simplify $i^2 = -1$)

Example: $2i(3+i) = 2(3i) + 2i(i)$ Distribute
 $= 6i + 2i^2$ Simplify
 $= 6i + 2(-1)$ Substitute
 $= -2 + 6i$ Simplify and rewrite in complex form

Since $i = \sqrt{-1}$, no answer can have an 'i' in the denominator. RATIONALIZE!

Simplify.

9b. $\sqrt{-49}$

10b. $6\sqrt{-12}$

11. $-6(2-8i) + 3(5+7i)$

12. $(3-4i)^2$

13. $(6-4i)(6+4i)$

Rationalize.

14. $\frac{1+6i}{5i}$

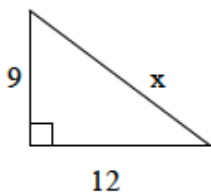
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Geometry:

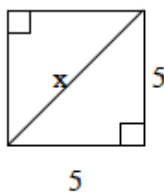
Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Find the value of x .

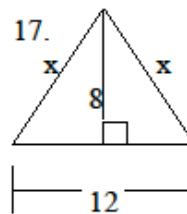
15.



16.

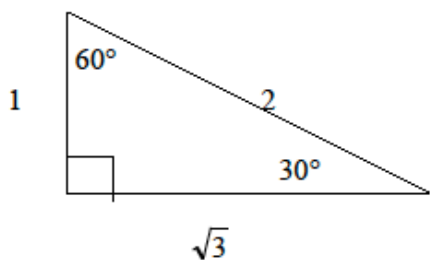


17.

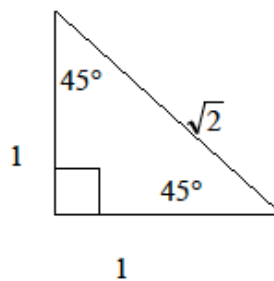


18. A square has perimeter 12 cm. Find the length of the diagonal.

* In $30^\circ - 60^\circ - 90^\circ$ triangles, sides are in proportion $1, \sqrt{3}, 2$.

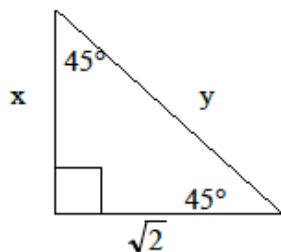


* In $45^\circ - 45^\circ - 90^\circ$ triangles, sides are in proportion $1, 1, \sqrt{2}$.

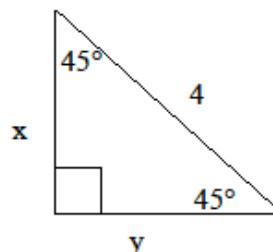


Solve for x and y .

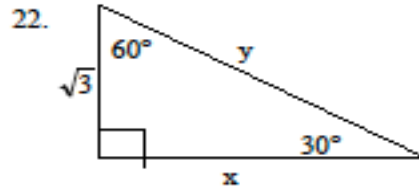
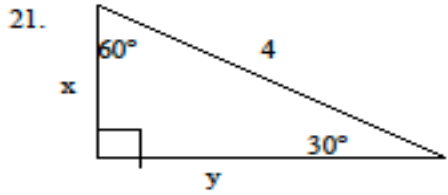
19.



20.



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Equations of Lines:

Slope-intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: $y = c$ (slope is zero)

Standard Form: $Ax + By = C$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equation: $5x - 4y = 8$

24. Find the x-intercept and y-intercept of the equation: $2x - y = 5$

25. Write the equation in standard form: $y = 7x - 5$

Write the equation of the line in slope-intercept form with the following conditions:

26. slope = -5 and passes through the point (-3, -8)

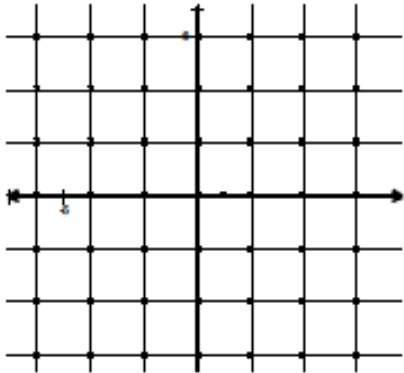
27. passes through the points (4, 3) and (7, -2)

28. x-intercept = 3 and y-intercept = 2

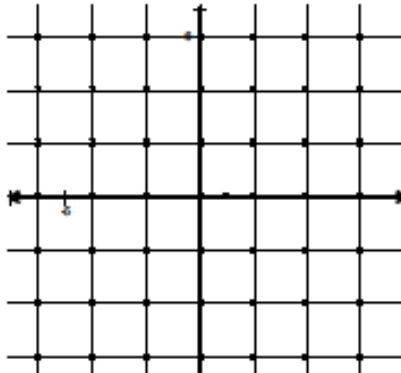
Graphing: Graph each function, inequality, and/or system.

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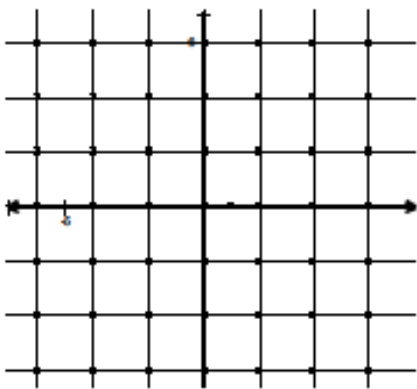
29. $3x - 4y = 12$



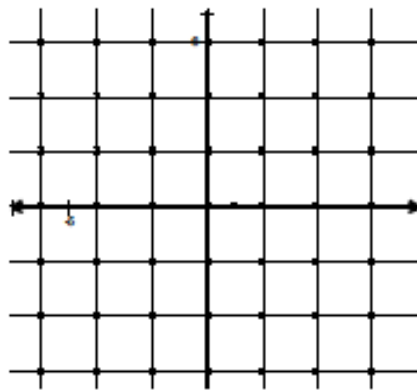
30. $\begin{cases} 2x + y = 4 \\ x - y = 2 \end{cases}$



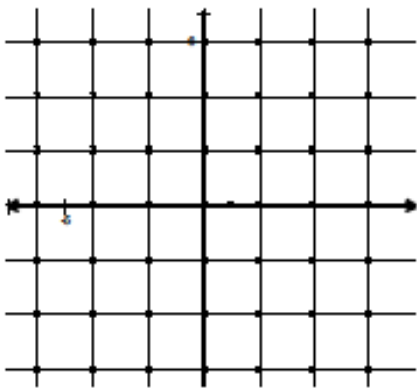
31. $y < -4x - 2$



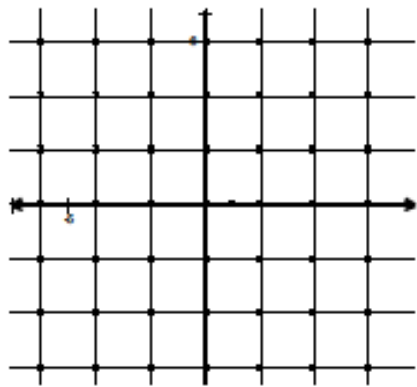
32. $y + 2 = |x + 1|$



33. $y > |x| - 1$



34. $y + 4 = (x - 1)^2$



Systems of Equations:

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$$\begin{cases} 3x + y = 6 \\ 2x - 2y = 4 \end{cases}$$

Substitution:

Solve 1 equation for 1 variable

Rearrange.

Plug into 2nd equation.

Solve for the other variable.

Elimination:

Find opposite coefficients for 1 variable

Multiply equation(s) by constant(s).

Add equations together (lose 1 variable)

Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$$y = 6 - 3x$$

Solve 1st equation for y

$$6x + 2y = 12$$

Multiply 1st equation by 2

$$2x - 2(6 - 3x) = 4$$

Plug into 2nd equation

$$\underline{2x - 2y = 4}$$

coefficients of y are opposite

$$2x - 12 + 6x = 4$$

Distribute

$$8x = 16$$

Add

$$8x = 16 \text{ and } x = 2$$

Simplify

$$x = 2$$

Simplify.

Plug x=2 back into the original equation $6 + y = 6$
 $y = 0$

Solve each system of equations, using any method.

35. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

36. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Exponents:

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Recall the following rules of exponents:

1. $a^1 = a$ Any number raised to the power of one equals itself.
2. $1^a = 1$ One raised to any power is one.
3. $a^0 = 1$ Any nonzero number raised to the power of zero is one.
4. $a^m \cdot a^n = a^{m+n}$ When multiplying two powers that have the same base, add the exponents.
5. $\frac{a^m}{a^n} = a^{m-n}$ When dividing two powers with the same base, subtract the exponents.
6. $(a^m)^n = a^{mn}$ When a power is raised to another power, multiply the exponents.
7. $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\frac{3c}{c^{-1}}$

40. $\frac{2ef^{-1}}{e^{-1}}$

41. $\frac{(n^3 p^{-1})^2}{(np)^{-2}}$

Simplify.

42. $3m^2 \cdot 2m$

43. $(a^3)^2$

44. $(-b^3 c^4)^5$

45. $4m(3a^2 m)$

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Polynomials:

To add/subtract polynomials, combine like terms.

EX: $8x - 3y + 6 - (6y + 4x - 9)$ *Distribute the negative through the parentheses.*
 $= 8x - 3y + 6 - 6y - 4x + 9$ *Combine like terms with similar variables.*
 $= 8x - 4x - 3y - 6y + 6 + 9$
 $= 4x - 9y + 15$

Simplify.

46. $3x^3 + 9 + 7x^2 - x^3$

47. $7m - 6 - (2m + 5)$

To multiply two binomials, use FOIL.

EX: $(3x - 2)(x + 4)$ *Multiply the first, outer, inner, and last terms.*
 $= 3x^2 + 12x - 2x - 8$ *Combine like terms together.*
 $= 3x^2 + 10x - 8$

Multiply.

48. $(3a + 1)(a - 2)$

49. $(s + 3)(s - 3)$

50. $(c - 5)^2$

51. $(5x + 7y)(5x - 7y)$

Factoring:

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Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

- a) If you have one (other than 1) factor it out.
- b) If you don't have one move on to STEP 2

STEP 2: How many terms does the polynomial have?

2 Terms a) is it the difference of two squares? $a^2 - b^2 = (a + b)(a - b)$

EX: $x^2 - 25 = (x + 5)(x - 5)$

b) Is it the sum or difference of two cubes? $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

EX: $m^3 + 64 = (m + 4)(m^2 - 4m + 16)$
 $p^3 - 125 = (p - 5)(p^2 + 5p + 25)$

3 Terms

EX:

$$x^2 + bx + c = (x + _)(x + _)$$

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

$$x^2 - bx - c = (x - _)(x - _)$$

$$x^2 - 5x + 4 = (x - 1)(x - 4)$$

$$x^2 + bx - c = (x - _)(x + _)$$

$$x^2 + 6x - 16 = (x - 2)(x + 8)$$

$$x^2 - bx - c = (x - _)(x + _)$$

$$x^2 - 2x - 24 = (x - 6)(x + 4)$$

4 Terms---Factor by Grouping

- a) Pair up first two terms and last two terms.
- b) Factor out GCF of each pair of numbers.
- c) Factor out front parentheses that the terms have in common.
- d) Put leftover terms in parentheses.

$$\begin{aligned} \text{Ex: } x^3 + 3x^2 + 9x + 27 &= (x^3 + 3x^2) + (9x + 27) \\ &= x^2(x + 3) + 9(x + 3) \\ &= (x + 3)(x^2 + 9) \end{aligned}$$

Factor completely.

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52. $z^2 + 4z - 12$

53. $6 - 5x - x^2$

54. $2k^2 + 2k - 60$

55. $-10b^4 - 15b^2$

56. $9c^2 + 30c + 25$

57. $9n^2 - 4$

58. $27z^3 - 8$

59. $2mn - 2mt + 2sn - 2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use the quadratic formula.

EX: $x^2 - 4x = 21$ *Set equal to zero FIRST.*

$x^2 - 4x - 21 = 0$ *Now factor.*

$(x + 3)(x - 7) = 0$ *Set each factor equal to zero.*

$x + 3 = 0$ $x - 7 = 0$ *Solve for each x.*

$x = -3$ $x = 7$

Solve each equation.

60. $x^2 - 4x - 12 = 0$

61. $x^2 + 25 = 10x$

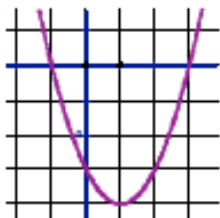
62. $x^2 - 14x + 40 = 0$

Discriminant: The number under the radical in the quadratic formula ($b^2 - 4ac$) can tell you what kind of roots you will have.

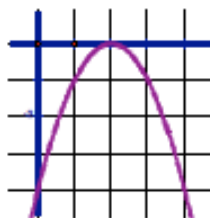
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If $b^2 - 4ac > 0$ you will have TWO real roots

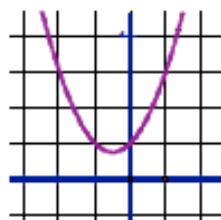
(touches the x-axis twice)



If $b^2 - 4ac = 0$ you will have ONE real root (touches axis once)



If $b^2 - 4ac < 0$ you will have TWO imaginary roots. (Function does not cross the x-axis)



QUADRATIC FORMULA—allows you to solve any quadratic for all its real and imaginary roots.

$$5x^2 - 2x + 4 = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EX: In the equation $x^2 + 2x + 3 = 0$, find the value of the discriminant, describe the nature of the roots, then solve.

$$x^2 + 2x + 3 = 0 \quad \text{Determine the values of } a, b, \text{ and } c.$$

$$a = 1 \quad b = 2 \quad c = 3 \quad \text{Find the discriminant.}$$

$$D = 2^2 - 4 \cdot 1 \cdot 3$$

$$D = 4 - 12$$

$$D = -8 \quad \text{There are two imaginary roots.}$$

$$\text{Solve: } x = \frac{-2 \pm \sqrt{-8}}{2}$$

$$x = \frac{-2 \pm 2i\sqrt{2}}{2}$$

$$x = -1 \pm i\sqrt{2}$$

Find the value of the discriminant, describe the nature of the roots, then solve each quadratic. Use EXACT values.

63. $x^2 - 9x + 14 = 0$

64. $5x^2 - 2x + 4 = 0$

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Discriminant = _____

Type of Roots: _____

Exact Value of Roots: _____

Discriminant = _____

Type of Roots: _____

Exact Value of Roots: _____

Long Division—can be used when dividing any polynomials.

Synthetic Division—can ONLY be used when dividing a polynomial by a linear polynomial.

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EX: $\frac{2x^3 + 3x^2 - 6x + 10}{x + 3}$

Long Division

$$\frac{2x^3 + 3x^2 - 6x + 10}{x + 3}$$

Synthetic Division

$$\frac{2x^3 + 3x^2 - 6x + 10}{x + 3}$$

$$\begin{array}{r}
 2x^2 - 3x + 3 + \frac{1}{x+3} \\
 x+3 \overline{) 2x^3 + 3x^2 - 6x + 10} \\
 \underline{(-)(2x^3 + 6x^2)} \\
 -3x^2 - 6x \\
 \underline{(-)(-3x^2 - 9x)} \\
 3x + 10 \\
 \underline{(-)(3x + 9)} \\
 1
 \end{array}$$

$$\begin{array}{r}
 2x^3 + 3x^2 - 6x + 10 \\
 \underline{ + 3x^2} \\
 -3 \quad 2 \quad 3 \quad -6 \quad 10 \\
 \quad -6 \quad 9 \quad -9 \\
 \quad 2 \quad -3 \quad 3 \quad 1 \\
 = 2x - 3x + 3 + \frac{1}{x+3}
 \end{array}$$

Divide each polynomial using long division OR synthetic division.

65. $\frac{c^3 - 3c^2 + 18c - 16}{c^2 + 3c - 2}$

66. $\frac{x^4 - 2x^2 - x + 2}{x + 2}$

To evaluate a function for the given value, simply plug the value into the function for x.

Evaluate each function for the given value.

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67. $f(x) = x^2 - 6x + 2$

$f(3) =$ _____

68. $g(x) = 6x - 7$

$g(x+h) =$ _____

69. $f(x) = 3x^2 - 4$

$5[f(x+2)] =$ _____

Composition and Inverses of Functions:

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read "**f of g of x**" means to plug the inside function in for x in the outside function.

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \end{aligned}$$

$$f(g(x)) = 2x^2 - 16x + 33$$

Suppose $f(x) = 2x$, $g(x) = 3x - 2$, and $h(x) = x^2 - 4$. **Find the following:**

70. $f[g(2)] =$ _____

71. $f[g(x)] =$ _____

72. $f[h(3)] =$ _____

73. $g[f(x)] =$ _____

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

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Example:	$f(x) = \sqrt[3]{x+1}$	Rewrite $f(x)$ as y
	$y = \sqrt[3]{x+1}$	Switch x and y
	$x = \sqrt[3]{y+1}$	Solve for your new y
	$(x)^3 = (\sqrt[3]{y+1})^3$	Cube both sides
	$x^3 = y+1$	Simplify
	$y = x^3 - 1$	Solve for y
	$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse, $f^{-1}(x)$, if possible.

74. $f(x) = 5x + 2$

75. $f(x) = \frac{1}{2}x - \frac{1}{3}$

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Multiplying and Dividing: Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.

EX: $\frac{x^2 + 10x + 21}{5 - 4x - x^2} \cdot \frac{x^2 + 2x - 15}{x^3 + 4x^2 - 21x}$ Factor everything completely.

$= \frac{(x + 7)(x + 3)}{(5 + x)(1 - x)} \cdot \frac{(x + 5)(x - 3)}{x(x - 3)(x + 7)}$ Cancel out common factors in the top and bottom.

$= \frac{(x + 3)}{x(1 - x)}$ Simplify.

76. $\frac{5z^3 + z^2 - z}{3z}$

77. $\frac{m^2 - 25}{m^2 + 5m}$

78. $\frac{10r^5}{21s^2} \cdot \frac{3s}{5r^3}$

79. $\frac{a^2 - 5a + 6}{a + 4} \cdot \frac{3a + 12}{a - 2}$

80. $\frac{6d - 9}{5d + 1} \div \frac{6 - 13d + 6d^2}{15d^2 - 7d - 2}$

Addition and Subtraction

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First find the least common denominator. Write each fraction with that LCD. Add/subtract numerators as indicated and leave the denominators as they are.

EX: $\frac{3x+1}{x^2+2x} + \frac{5x-4}{2x+4}$ *Factor denominator completely.*

$\frac{3x+1}{x(x+2)} + \frac{5x-4}{2(x+2)}$ *Find LCD, which is $(2x)(x+2)$*

$\frac{2(3x+1)}{2x(x+2)} + \frac{x(5x-4)}{2x(x+2)}$ *Rewrite each fraction with the LCD in the denominator.*

$\frac{6x+2+5x^2-4x}{2x(x+2)}$ *Write as one fraction.*

$\frac{5x^2+2x+2}{2x(x+2)}$ *Combine like terms.*

81. $\frac{2x}{5} - \frac{x}{3}$

82. $\frac{b-a}{a^2b} + \frac{a+b}{ab^2}$

83. $\frac{2-a^2}{a^2+a} + \frac{3a+4}{3a+3}$

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Complex Fractions: Eliminate complex fractions by multiplying the numerator and denominator by the LCD of each of the small fractions. Then simplify the result.

EX:
$$\frac{1 + \frac{1}{a}}{\frac{2}{a^2} - 1}$$

Find LCD: a^2

$$= \frac{\left(1 + \frac{1}{a}\right) \cdot a^2}{\left(\frac{2}{a^2} - 1\right) \cdot a^2}$$

Multiply top and bottom by LCD.

$$= \frac{a^2 + a}{2 - a^2}$$

Factor and simplify if possible.

$$= \frac{a(a+1)}{2 - a^2}$$

84.
$$\frac{1 - \frac{1}{2}}{2 + \frac{1}{4}}$$

85.
$$\frac{1 + \frac{1}{z}}{z + 1}$$

86.
$$\frac{5 + \frac{1}{m} - \frac{6}{m^2}}{\frac{2}{m} - \frac{2}{m^2}}$$

87.
$$\frac{2 + \frac{1}{x} - \frac{1}{x^2}}{1 + \frac{4}{x} + \frac{3}{x^2}}$$

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Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of our fractions. Then solve the equation as usual.

$$\frac{5}{x+2} + \frac{1}{x} = \frac{5}{x}$$

Find LCD first $x(x+2)$

$$x(x+2)\frac{5}{x+2} + x(x+2)\frac{1}{x} = \frac{5}{x}x(x+2)$$

Multiply each term by the LCD.

$$5x + 1(x+2) = 5(x+2)$$

Simplify and solve.

$$5x + x + 2 = 5x + 10$$

$$6x + 2 = 5x + 10$$

$$x = 8 \quad \leftarrow \text{Check your answer! Sometimes they do not check!}$$

Check: $\frac{5}{8+2} + \frac{1}{8} = \frac{5}{8}$

$$\frac{5}{10} + \frac{1}{8} = \frac{5}{8}$$

$$\frac{5}{8} = \frac{5}{8}$$

Solve each equation. Check your solutions.

88. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$

89. $\frac{x+10}{x^2-2} = \frac{4}{x}$

90. $\frac{5}{x-5} = \frac{x}{x-5} - 1$

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Solutions to Odd Exercises

revised 5/11

1. $4\sqrt{2}$ 3. -4 5. $\frac{\sqrt{11}}{3}$ 7. $5 + \sqrt{10} - \sqrt{30} - 2\sqrt{3}$ 9a. $\frac{2\sqrt{3}}{3}$

9b. $7i$ 11. $3 + 69i$ 13. 52 15. 15 17. 10 19. $\sqrt{2}$

21. $x=2$ $y=2\sqrt{3}$ 23. $m = \frac{5}{4}$ $b = -2$ 25. $7x - y = 5$ 27. $y = -\frac{5}{3}x + \frac{29}{3}$

29, 31, 33 graphs 35. $(7, -10)$ 37. $\left(\frac{89}{36}, -\frac{29}{18}\right)$ 39. $3c^2$ 41. n^8

43. a^6 45. $12a^2m^2$ 47. $5m - 11$ 49. $s^2 - 9$

51. $25x^2 - 49y^2$ 53. $-(x+6)(x-1)$ 55. $-5b^2(2b^2 + 3)$ 57. $(3n-2)(3n+2)$

59. $2(m+s)(n-t)$ 61. $x = 5$ 63. 25 ; 2 real; 7 and 2 65. $(c-6) + \frac{38c-28}{c^2+3c-2}$

67. -7 69. $15x^2 + 60x + 40$ 71. $6x - 4$ 73. $6x - 2$

75. $f^{-1}(x) = 2x + \frac{2}{3}$ 77. $\frac{m-5}{m}$ 79. $3(a-3)$ 81. $\frac{x}{15}$ 83. $\frac{4a+6}{3a(a+1)}$

85. $\frac{1}{z}$ 87. $\frac{2x-1}{x+3}$ 89. $x = 4, -\frac{2}{3}$